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Nijenhuis Geometry

Lecture Course

This course is an introduction to Nijenhuis Geometry, a new challenging area in Differential Geometry that studies local and global properties of geometric structures given by a field of endomorphisms with vanishing Nijenhuis torsion. This topic is located on the crossroad of Geometry, Mathematical Physics and Algebra as Nijenhuis structures naturally appear in many seemingly unrelated research areas such as bi-Hamiltonian integrable systems (both finite and infinite-dimensional), projective geometry, theory of left-symmetric algebras and others.

The course will be based on a series of recent papers/preprints:

- (1) A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, *Nijenhuis Geometry*, arXiv:1903.04603 (accepted by Advances in Math.)
- (2) A. Yu. Konyaev, *Nijenhuis Geometry II: left-symmetric algebras and linearization problem*, Diff. Geom. and Appl. 74 (2021) 101706, arXiv:1903.06411.
- (3) A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, *Nijenhuis Geometry III: gl -regular Nijenhuis operators*, arXiv:2007.09506 (submitted to Jour. of London Math. Soc.)
- (4) A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, *Applications of Nijenhuis Geometry: Non-degenerate singular points of Poisson-Nijenhuis structures*, European J. Math. (2021) arXiv:2001.04851.
- (5) A. V. Bolsinov, A. Yu. Konyaev, V. S. Matveev, *Applications of Nijenhuis geometry II: maximal pencils of multihamiltonian structures of hydrodynamic type*, arXiv:2009.07802 (submitted to Nonlinearity)
- (6) A. Bolsinov, V. Matveev, E. Miranda, S. Tabachnikov, *Open problems, questions, and challenges in finite-dimensional integrable systems*, Phil. Trans. R. Soc. A 376 (2018), 20170430, arXiv:1804.03737.

Programme

- Fields of endomorphisms. Nijenhuis torsion and Nijenhuis operators: equivalent definitions.
- Basic properties of Nijenhuis operators. Splitting theorem.
- Diagonalisable and differentially non-degenerate Nijenhuis operators.
- Nijenhuis operators with complex eigenvalues. Generalised Nirenberg-Newlander theorem.
- Nilpotent Nijenhuis operators and Jordan blocks.
- Singular points of Nijenhuis operators and linearisation.
- Left-symmetric algebras. Linearisability and non-degeneracy.
- gl -regular Nijenhuis operators and their canonical forms.
- Nijenhuis perturbations of a Jordan block.
- Normal forms for gl -regular Nijenhuis operators in dimension 2.
- Global properties of Nijenhuis operators on closed manifolds.
- Nijenhuis operators and bi-Hamiltonian systems.
- Nijenhuis operators and geodesically equivalent metrics.
- Nijenhuis operators and Poisson brackets of hydrodynamic type.
- Open problems in Nijenhuis Geometry.