INTERNATIONAL WORKSHOP

"PROBABILITY, ANALYSIS AND GEOMETRY"

(*Lomonosov Moscow State University and Ulm University*)

Moscow, September 30 – October 4, 2014

ABSTRACTS

On the entropy of the directional distribution in fiber systems

Patricia Alonso Ruiz (*Ulm University*, PROBABILITY, 30 min)

In this talk we investigate a germ-grain process usually employed as simple model of fiber system. Such objects are often used to describe the structure plastics commonly used in industry such as polyamides. In particular we consider an homogeneous Poisson marked process where the marks represent the direction of the fibers and study some properties of the Kolmogorov entropy of the directional distribution's density.

This work is supported by the DAAD, Strategic Partnership Program.

Local solutions to a class of Monge-Ampère equations of mixed type

Lukas Bartholomäus (*Ulm University*, ANALYSIS, 30 min)

The existence of a graph $\{(x, u(x))|\ x \in U \subset \mathbb{R}^n,\ u \in C^2(U)\}$ with a given Gauss curvature K is equivalent to the solvability of the Monge-Ampère equation

$$\det(D^2u) = K(1 + |Du|^2)^{(n+2)/2}.$$

We study a class of Monge-Ampère equations of mixed type which are elliptic on one side of a hypersurface and hyperbolic on the opposite side. We are trying to establish the existence of sufficiently smooth local solutions of the Monge-Ampère equation via Nash-Moser iteration.

An important step of the proof is the construction of a family of local, ++regularizing operators $(S_l)_{l\in\mathbb{N}_0},\ S_l : L^2(\Omega) \to C^\infty(\Omega)$, where $\Omega \subset U$ lies on one side of the flattened hypersurface.

This work is supported by the DAAD, Strategic Partnership Program.

Computational aspects of L-functions

Michel Börner (*Ulm University*, GEOMETRY, 20 min)

In our talk we will describe what it means to "compute" an L-function. Furthermore, we will report on concrete caculations of the Hasse-Weil L-function of some curves of genus >= 2 over the rationals, and our attempts to verify their functional equation.

This work is supported by the DAAD, Strategic Partnership Program.

Lower bounds for the Kantorovich distance and multidimensional analogs of the Hardy–Landau–Littlewood inequality

V.I. Bogachev (*Lomonosov Moscow State University*, ANALYSIS, 45 min)

The classical inequality of Hardy–Landau–Littlewood estimates the square of the L^1-norm of the first derivative of a function via the product of the L^1-norms of the function itself and its second derivative. This estimate can be written as a bound on the variation distance between two probability measures with absolutely continuous densities via the Kantorovich distance between them and the variation of the difference of these densities. This talk based on a joint paper with Alexander Shaposhnikov will be concerned with a multidimensional generalization of the latter estimate, which provides lower bounds for the Kantorovich distance between multidimensional distributions in terms of Sobolev and BV norms of their densities. Similar estimates will be also discussed for measures on Riemannian manifolds.

Density of semigroups in Banach spaces

P.A. Borodin (*Lomonosov Moscow State University*, ANALYSIS, 45 min)

The conditions on a set M in a Banach space X are searched, which are necessary and sufficient for the set $R(M)$ of sums $x_1 + \cdots + x_n$, $x_k \in M$, to be dense in X. In particular it was proved that if M is a rectifiable curve in a uniformly rotund and uniformly smooth Banach space X, this curve M does not lie in any semispace $\{x \in X : f(x) \geq 0\}$ ($f \in X^*$) ad is minimal in the sense that every its proper subarc lies in some open semispace $\{x \in X : f(x) > 0\}$, then $\overline{R(M)} = X$. The obtained results are applied to approximations in different functional spaces.

Fiberwise multiplicative Hirzebruch genera

V.M. Buchstaber (*Steklov Mathematical Institute of RAS, Lomonosov Moscow State University*, GEOMETRY, 50 min)

The theory of Hirzebruch genera of manifolds is a well-known area of algebraic topology. It has important applications in the theory of differential operators on manifolds, mathematical physics and combinatorics.

The lecture is devoted to results on Hirzebruch genera of smooth fiber bundles whose fibers are smooth manifolds with action of compact torus. In the focus of our interest will be the construction and solution of functional equations arising from the theorem of localization of the universal toric Hirzebruch genus for homogeneous spaces of compact Lie groups.

Various approaches to the identification of significant factors

A.V. Bulinski (*Lomonosov Moscow State University*, PROBABILITY, 45 min)

In many stochastic models a real-valued random response variable Y depends on some (in general random) factors $X_1, \ldots, X_n$. A very important problem is to determine the collection of indices $(k_1, \ldots, k_r)$ where $1 \leq k_1 < \ldots < k_r \leq n$ such that Y depends "essentially" on $(X_{k_1}, \ldots, X_{k_r})$ and the complementary set of factors X_i with $i \notin \{k_1, \ldots, k_r\}$ can be discarded as negligible. There are different approaches to solution of this problem. We discuss diverse statistical methods employed here such as principle component analysis, logic and logistic regressions, LASSO and various machine learning techniques. The development of MDR (multifactor dimensionality reduction) method is treated in more detail. In this regard we refer to quite recent papers with A.S.Rakitko.

Arithmetic and harmonic analysis

V.N. Chubarikov (*Lomonosov Moscow State University*, ANALYSIS, 50 min)

The lecture is devoted to applications of trigonometric sums and integrals to problems of arithmetics and analysis. Special attention is paid to additive problems of number theory, the Laplace-Beltrami equation, the Shroedinger equation as well as to problems related to very short sums of periodical arithmetic functions.

Elastic plates and surfaces

Anna Dall'Acqua (*Ulm University*, ANALYSIS, 50 min)

Equilibrium positions of elastic plates are solutions of fourth order elliptic boundary value problems. The methods developed to treat second order elliptic equations based on comparison principles are in general not available any more. In the lecture we discuss this issue and present some existence results concerning elastic surfaces.

This work is supported by the DAAD, Strategic Partnership Program.

Legendrian links, monotonic simplification, and Jones' conjecture

I.A. Dynnikov (*Steklov Mathematical Institute of RAS,*
Lomonosov Moscow State University, GEOMETRY, 50 min)

I will overview our joint work with Maxim Prasolov in which we prove that a rectangular diagram admits a monotonic simplification by elementary moves if and only if at least one of the two Legendrian links associated to the diagram admits a destabilisation. As a corollary we obtain Jones' conjecture that states that two braids representing the same oriented link type and having minimal braid index have the same writhe. Also, we get a method to compute maximal Thurston-Bennequin number for any link type.

Parameter estimation of Lévy-driven CARMA processes

Zywilla Fechner (*Ulm University*, PROBABILITY, 30 min)

The aim of this talk is to discuss estimation procedures for Lévy driven CARMA models. Most of the estimators for this model are based on observations made on an equidistant discrete grid. The aim of our research is to find a suitable estimator for unevenly-spaced high-frequency observations. We are going to discuss some classical results for a non-equidistant discrete grid and present some results for high-frequency data based on frequency-domain methods, in particular on a Whittle-type estimation procedure applied for the periodogram of a CARMA process.

This work is supported by the DAAD, Strategic Partnership Program.

The stability of orthogonal greedy expansions

N.N. Fedotov (*Lomonosov Moscow State University*, ANALYSIS, 20 min)

The model of orthogonal greedy expansion that allows considering projection errors will be presented, and conditions on the projection errors and non-optimal selection of expanding elements that guarantee the convergence of orthogonal greedy approximations exactly to the approximated element will be discussed.

On locally and bounded Chebyshev sets

A.A. Flerov (*Lomonosov Moscow State University*, ANALYSIS, 20 min)

We extend the notion of a locally Chebyshev set and study connections between Chebyshev and locally Chebyshev sets. Then we obtain the following result: a connected closed locally Chebyshev set in a two-dimensional Banach space is also Chebyshev, and each Chebyshev set is locally Chebyshev if and only if space is strictly convex.

The intermediate case of regularity in the problem of differentiation of multiple integrals

D. V. Fufaev (*Lomonosov Moscow State University*, ANALYSIS, 20 min)

The talk will be about the generalization of Lebesgue and Jessen-Marcinkiewicz-Zygmund theorems of the differentiation of multiple integrals for the intermediate case of regularity of the system of sets. The application of the result to the Fourier-Haar series and to orthorecursive expansions with respect to system of indicators of parallelograms is considered.

On integral cohomology ring of symmetric products

D.V. Gugnin (*Lomonosov Moscow State University*, GEOMETRY, 30 min)

Let us denote by $\mathrm{Sym}^n X$ the n-th symmetric product X^n/S_n of a topological space X. The famous result of A.Dold states that if two connected CW complexes X and Y has equal integral homology $H_i(X;\mathbb{Z}) = H_i(Y;\mathbb{Z}), 1 \leq i \leq q$, then $H_i(\mathrm{Sym}^n X;\mathbb{Z}) = H_i(\mathrm{Sym}^n Y;\mathbb{Z}), 1 \leq i \leq q$, for all $n > 1$.

From this moment suppose that all spaces X has finitely generated integral homology in each dimension. Then from A.Dold's result it is easy to check that integral cohomology $H^*(\mathrm{Sym}^n X;\mathbb{Z})$ is also finitely generated in each dimension. Then one has ring isomorphisms

$$H^*(\mathrm{Sym}^n X;\mathbb{Z}) \otimes \mathbb{Q} \cong (H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor}) \otimes \mathbb{Q} \cong H^*(\mathrm{Sym}^n X;\mathbb{Q}).$$

If for two given CW complexes X and Y their rational cohomology rings are equal, $H^*(X;\mathbb{Q}) = H^*(Y;\mathbb{Q}) = A^*$, then by simple classical Transfer Theorem one has equality $H^*(\mathrm{Sym}^n X;\mathbb{Q}) = H^*(\mathrm{Sym}^n Y;\mathbb{Q}) = S^n A^*$, where $S^n A^* := (A^{\otimes n})^{S_n}$.

One of the main results of my talk is the following theorem.

Theorem 1(G., 2014). *Let X and Y be connected CW complexes s.t. $H^*(X;\mathbb{Z})/\mathrm{Tor} \cong H^*(Y;\mathbb{Z})/\mathrm{Tor}$. Then there exists an isomorphism of rings*

$$H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor} \cong H^*(\mathrm{Sym}^n Y;\mathbb{Z})/\mathrm{Tor}$$

for all $n > 1$. Moreover, for any additive basis

$$a_{i,j} \in H^i(X;\mathbb{Z})/\mathrm{Tor}, i > 0, 1 \leq j \leq \mathrm{rank}(H^i(X;\mathbb{Z})/\mathrm{Tor}),$$

and integral multiplication table $a_{i,j}a_{k,l} = c^{s,t}_{i,j;k,l}a_{s,t}$, there is an explicit algorithm for constructing some additive basis of $H^*(\mathrm{Sym}^n X;\mathbb{Z})/\mathrm{Tor}, n > 1$, and computing the multiplication table for this basis.

Suppose $g > 0$. The famous result of I.G.Macdonald states the following.

Theorem (I.G.Macdonald,1962). *Suppose M^2_g is an arbitrary compact Riemann surface of genus $g > 0$. Then the ring $H^*(\mathrm{Sym}^n M^2_g;\mathbb{Z})$ has no torsion and is isomorphic to free graded commutative algebra over $\mathbb{Z}$ on $2g$ generators of degree 1 and one generator of degree 2 factorized by some concrete integral relations (I.G.Macdonald's ideal).*

But, as was noticed by the author in 2012, the proof of I.G.Macdonald's theorem relies on some proposition, which in general is false. However, in the special case of I.G.Macdonald's theorem, this proposition turns out to be true. The verification of the I.G.Macdonald theorem, which follows from Theorem 1 above and additional reasoning, is the second main result to be presented on my talk.

Higher order orbifold Euler characteristics and their generalizations

S.M. Gusein-Zade (*Lomonosov Moscow State University*, GEOMETRY, 45 min)

For a "good enough" topological space with a finite group action, higher order orbifold Euler characteristics are generalizations of the orbifold Euler characteristic introduced by physicists. One has computed the generating series of the higher order Euler characteristics of a fixed order of the Cartesian products of the manifold with the wreath product

actions on them (getting formulae of MacDonald type). We discuss some generalizations of these notions. They include their motivic versions (with values in the Grothendieck ring of complex quasi-projective varieties extended by the rational powers of the class of the affine line) and their versions for compact Lie group actions. We give formulae for the generating series of these generalized Euler characteristics for the wreath product actions.

The talk is based on joint works with I. Luengo and A. Melle-Hernndez.

Maximal non-exchangeability: consequences and tests

Michael Harder* and Ulrich Stadtmüller
(*Ulm University*, PROBABILITY, 30 min)

Studying the dependence structure in the distribution function H of a d-dimensional continuous random vector $\mathbf{X}$, the so called copula is crucial. Without being independent, $\mathbf{X}$ (i.e. its components) might still be exchangeable. Exchangeability is a property of the copula. When choosing a family of copulas in order to model a given situation, it is important to know whether an exchangeable copula fits the data.

In this talk, a measure for the absence of exchangeability is presented. At first, explicit bounds for this measure will be given, generalizing a result for dimension two by Nelsen (2007), and Klement and Mesiar (2006). Second, two-dimensional testing procedures by Genest et al. (2012) will be generalized for arbitrary dimension.

This work is supported by the DAAD, Strategic Partnership Program.

On the problem of Gaussian optimizers in quantum information theory

A.S. Holevo (*Steklov Math. Institute of RAS*, PROBABILITY, 45 min)

In classical analysis a result is known, which briefly sounds as follows: "Gaussian kernels have only Gaussian maximizers" (Lieb, based on the work of K.I. Babenko, Beckner, Carlen, Nelson and others). The point is that the norm of a Gaussian integral operator from L_p to L_q, under certain conditions, is attained (only) on a Gaussian function.

A noncommutative analog of such an integral operator is bosonic Gaussian channel, which is a completely positive map F of the algebra of canonical commutation relations. Recently the long-standing conjecture of quantum Gaussian optimizers has been resolved by showing that the spectrum of the image of any state under the mapping F is majorized by the spectrum of the image of a coherent state (pure Gaussian quantum state), and coherent states are characterized by this property. This enables optimization for the Schatten norms, output Renyi entropies and von Neumann entropies of a quantum Gaussian map F, which allows us to give explicit expressions for the communication capacity of bosonic Gaussian channels, including those widely used in quantum optics.

Minimal filings of finite metric spaces

A.O. Ivanov (*Lomonosov Moscow State University*, GEOMETRY, 45 min)

We present a review on a new branch of one-dimensional geometrical optimization problem, the minimal fillings theory. This theory is connected closely with generalized Steiner problem and gives an opportunity to look at many classical questions appearing in optimal connection theory from new point of view.

Central limit theorems for geometric functionals of random fields

Jürgen Kampf (*Ulm University*, PROBABILITY, 30 min)

This talk is based on work in progress. In this talk we are interested in central limit theorems for random fields, as the observation window approaches the whole Eulidean space. Such central limit theorems are already known for the Lebesgue measure of excursion sets. Now we consider, more generally, Lebesgue integrals of the random field f(X(t)), where X is the original random field and f is a fixed deterministic function. We will indicate how statistical applications of these theorems may look like.

This work is supported by the DAAD, Strategic Partnership Program.

Large gaps between consecutive prime numbers

S.V. Konyagin (*Steklov Math. Institute of RAS, Lomonosov Moscow State University*, ANALYSIS, 45 min)

In this talk we discuss the results of quite recent paper by K. Ford, B. Green, the speaker, and T. Tao.

Let $G(x)$ denote the size of the largest gap between consecutive primes below x. Answering a question of Erdős, we show that

$$G(x) \geq f(x)\frac{\log x \log\log x \log\log\log\log x}{(\log\log\log x)^2},$$

where $f(x)$ is a function tending to infinity with x.

On group factor isomorphism problem

A.I. Korchagin (*Lomonosov Moscow State University*, GEOMETRY, 20 min)

In this talk we consider an isomorphism of L_2 and L_∞, where L_n is von Neumann group factor of free group with n generators. We try to construct countable family of free Haar unitaries in L_2. Voiculescu's noncommutative probability methods will be irreplaceable helper.

Product representations for random variables with Weibull distributions and their applications

V.Yu. Korolev (*Lomonosov Moscow State University*, PROBABILITY, 45 min)

In the communication, product representations are presented for random variables with the Weibull distribution in terms of random variables with normal, exponential and stable distributions yielding scale mixture representations for the corresponding distributions. Main attention is paid to the case where the shape parameter γ of the Weibull distribution belongs to the interval $(0, 1]$. The case of small values of γ is of special interest, since the Weibull distributions with such parameters occupy an intermediate position between distributions with exponentially decreasing tails (e. g., exponential and gamma distributions) and heavy-tailed distributions with Zipf–Pareto power-type decrease of tails. It is demonstrated that if $\gamma \in (0, 1]$, then the Weibull distribution is a mixed half-normal law, and hence, it can be limiting for maximal random sums of independent random variables with finite variances. It is also demonstrated that the symmetric two-sided Weibull distribution with $\gamma \in (0, 1]$ is a scale mixture of normal laws. Necessary and sufficient conditions are presented for the convergence of the distributions of extremal random sums of independent random variables with finite variances and of the distributions of the absolute values of these random sums to the Weibull distribution as well as of those of random sums themselves to the symmetric two-sided Weibull distribution. These results can serve as theoretical grounds for the application of the Weibull distribution as an asymptotic approximation for statistical regularities observed in the scheme of stopped random walks used, say, to describe the evolution of stock prices and financial indexes. Also, necessary and sufficient conditions for the convergence of the distributions of more general regular statistics constructed from samples with random sizes to the symmetric two-sided Weibull distribution are discussed.

Estimates of integral norms of polynomials on spaces with convex measures

E.D. Kosov (*Lomonosov Moscow State University*, ANALYSIS, 20 min)

We discuss polynomials on spaces with convex measures and some related integral inequalities. In particular, our results show that any measurable polynomial with respect to a convex measure is integrable to every positive power and all L^p-norms are equivalent on the space of polynomials of a fixed degree. Moreover, there are estimates of L^1-norms of polynomials by L^1-norms of their restrictions to any set of positive measure.

Discrete invariants and topology of the Kovalevskaya integrable cases on the Lie algebras so(4) and so(3,1)

I.K. Kozlov (*Lomonosov Moscow State University*, GEOMETRY, 30 min)

In the talk we study the topological properties and some discrete invariants of integrable Hamiltonian systems on the Lie algebras so(4) and so(3, 1), which can be regarded as analogues of the Kovalevskaya integrable case in rigid body dynamics. In particular we construct the bifurcation diagrams of the momentum mapping, determine types of singularities and calculate the FomenkoZieschang invariants for the isoenergetic surfaces with trivial area constant.

System redundancy and generalizations of orthogonal expansions

T.P. Lukashenko, V.V. Galatenko (*Lomonosov Moscow State University*, ANALYSIS, 45 min)

The generalizations of orthogonal expansions including Parseval frames and orthorecursive expansions will be considered. These generalizations have certain classical properties of orthogonal expansions, but they are able to utilize redundant systems. The benefits of expansions in redundant systems, such as error detection and absolute stability with respect to wide classes of computational errors will be discussed.

High degree vertices in the preferential attachment model with choice

Yu.A. Malyshkin (*Lomonosov Moscow State University*, PROBABILITY, 20 min)

We prove almost sure convergence of the first k-th highest degrees in an evolving tree model combining local choice and preferential attachment. At each step in the growth of the graph, a new vertex is introduced. Then, we connect it with the one of d (with $d > 2$) possible neighbors, which are sampled from the existing vertices with probability proportional to degree. Of these possibilities, the vertex with the largest degree is chosen. The highest degrees in this model has linear behavior. This contrasts sharply with what is seen in the same choice model without preferential attachment.

On the long time behavior of some multidimensional stochastic synchronization systems

A.D. Manita (*Lomonosov Moscow State University*, PROBABILITY, 30 min)

We propose stochastic N-component synchronization models $(x_1(t), \dots, x_N(t))$, $x_j \in \mathbb{R}^d$, $t \in \mathbb{R}_+$, whose dynamics is described by Lévy processes and synchronizing jumps. We prove that symmetric models reach synchronization in a stochastic sense: differences between components $d_{kj}^{(N)}(t) = x_k(t) - x_j(t)$ have limits in distribution as $t \to \infty$. We give conditions of existence of natural (intrinsic) space scales for large synchronized systems, i.e., we are looking for such sequences $\{b_N\}$ that distribution of $d_{kj}^{(N)}(\infty)/b_N$ converges to some limit as $N \to \infty$. It appears that such sequence exists if the Lévy process enters a domain of attraction of some stable law. For Markovian synchronization models based on α-stable Lévy processes this results holds for any finite N in the precise form with $b_N = (N-1)^{1/\alpha}$. For non-Markovian models similar results hold only in the asymptotic sense. The class of limiting laws includes the Linnik distributions. We also discuss generalizations of these theorems to the case of non-uniform matrix-based intrinsic scales. The central point of our proofs is a representation of characteristic functions of $d_{kj}^{(N)}(t)$ via probability distribution of a superposition of N independent renewal processes.

On n-term approximations with respect to frames bounded in $L^p(0,1), 2 < p \leq \infty$

A.V. Meleshkina (*Lomonosov Moscow State University*, ANALYSIS, 20 min)

Best canonical n-term approximations in the norm of the spaces $L^2(0,1)$ of the family $\mathbb{I}$ of characteristic functions of intervals are studied. We got the estimate for an arbitrary tight $\Phi = \{\varphi_k\}_{k=1}^{\infty} \subset L^2(0,1)$ satisfying condition $\|\varphi_k\|_{L^p(0,1)} \leq D,\ k \in \mathbb{N},\ 2 < p \leq \infty$.

Cohomology of Landweber-Novikov algebra and singular Virasoro vectors

D.V. Millionshchikov (*Lomonosov Moscow State University*, GEOMETRY, 30 min)

We consider the group of substitutions of formal power series

$$\varphi(t) = t + x_1 t^2 + x_2 t^3 + x_3 t^4 + \ldots, \quad x_i \in Z, i = 1, 2, 3, \ldots$$

It was the fundamental observation by Buchstaber and Shokurov in 70s that the ring of left-invariant differential operators on this group is isomorphic to the Landweber-Novikov algebra in the complex cobordisms theory and its tensor product by reals is isomorphic to the universal enveloping algebra $U(L_1)$ of the Lie algebra of formal vector fields on the real line which vanish at the zero together with their first derivative.

We consider Feigin-Fuchs-Wallach-Rocha-Carridi free resolution of a one-dimensional trivial $U(L_1)$-module. For a long time the formulas for differentials of this resolution were unknown. We present the explicit answer for this question in terms of so-called Virasoro singular vectors and discuss applications to cohomology calculations.

Limit theorems for non Hermitian random matrices

A.A. Naumov (*Lomonosov Moscow State University*, PROBABILITY, 30 min)

One of the main questions studied in Random Matrix Theory is the asymptotic universality of the distribution of spectra of random matrices when their dimension goes to in?nity. The universality means the dependence on a few global characteristics of the distribution of the matrix entries. This holds, for example, for the spectra of Hermitian random matrices with independent entries (up to symmetry), and the spectrum of non Hermitian random matrices with independent identically distributed entries. In this talk we show the asymptotic universality of the distribution of spectra of random matrices with correlated entries. In this case the limiting distribution is given by Girkos elliptic law. We also study the product of such matrices and show that the limiting distribution for spectra doesnt depend on correlation between matrix entries and is given by the m-th power of random variable uniformly distributed on the unit circle. The talk is based on the joint work with F. Goetze and A. Tikhomirov.

Theory of topological invariants for integrable systems: how it works

S.S. Nikolaenko (*Lomonosov Moscow State University*, GEOMETRY, 20 min)

In this talk I would like to concentrate on some applications of the theory of topological invariants for the integrable Hamiltonian systems with two degrees of freedom created by A. T. Fomenko, H. Zieschang, A. V. Bolsinov and others to the study of new cases of Liouville and orbital equivalence between some classical mechanical systems. In particular, the integrable cases of Chaplygin and Goryachev will be discussed.

Weak parities and functorial maps

I.M. Nikonov (*Lomonosov Moscow State University*, GEOMETRY, 30 min)

Knots can be considered as equivalence classes of diagrams modulo Reidemeister moves. Functorial maps on knots which were introduced by V.O. Manturov, are consistent transformations of diagrams that turn equivalent diagrams to equivalent ones. Functorial maps can be described by compatible decorations of vertices of diagrams with 0s and 1s(weak parities). We discuss some properties of functorial maps and classify them for some knot theories.

Laplace transform for heat equation with pertrubated order of derivative

G.V. Nosovskiy (*Lomonosov Moscow State University*, GEOMETRY, 30 min)

Laplace transform is standard method for PDE with constant fractional derivatives. But applicability of LT for PDE with time-changing derivative order was an open problem.

Commuting differential operators of rank 2

V. Oganesyan (*Lomonosov Moscow State University*, GEOMETRY, 20 min)

Consider differential operators

$$L_n = \sum_{n=0}^{n} u_i(x)\partial_x^i, L_m = \sum_{i=0}^{m} v_i(x)\partial_x^i$$

The task of finding commuting differential operators is a classical problem of differential equations. Its theory has many applications. For instance stationary KdV equation $u_{xxx} - 6uu_x = 0$ is equivalent to condition of commutativity of two operators $L_2 = \partial_x^2 + u$ and $L_3 = \partial_x^3 + \frac{3}{2}u\partial_x + \frac{3}{4}u'$. If two ordinary differential operators commute then there is a nontrivial polynom $R(z, w)$ such that $R(L_n, L_m) = 0$. The curve $R(z, w) = 0$ is called the spectral curve. The number of linear independent solutions of

$$\begin{cases} L_n\psi = z\psi, \\ L_m\psi = w\psi \end{cases}$$

is called the rank of two commuting operators L_n, L_m. If its rank equals to one then there are explicit formulas in terms of Riemann theta-functions. The case of rank greater then one is much more difficult. In this talk we will consider new operators of rank 2.

Some possible numbers of edge coverings of a bipartite graph or shortest paths with fixed ends in a space of compact sets in $\mathbb{R}^n$

Z.N. Ovsyannikov (*Lomonosov Moscow State University*, GEOMETRY, 20 min)

Hausdorff distance was introduced in the beginning of XX century in order to measure distance between compact sets. In [1] numbers of shortest paths between points in $H(\mathbb{R}^n)$ were studied, a link with a graph theory was found and was shown that for any number from 1 to 36 (except 19) there is a pair of compact sets in $\mathbb{R}^n$ such that there is such a number of shortest paths between them. It was shown that there is no such pair for 19. Turns out, by using machine computation we can show there is no such pair of compact sets in $\mathbb{R}^n$ that there are 19, 37, 41, 59 or 67 shortest paths in $H(\mathbb{R}^n)$ between this two compacts.

Symmetry reduction and spectral geometry

A.V. Penskoi (*Lomonosov Moscow State University*, GEOMETRY, 45 min)

The question of description of metrics extremal for eigenvalues of Laplace operator is one of the classical questions of spectral geometry. Recently it turned out that the extremal metrics are exactly the metrics on minimal submanifolds in spheres. The condition of minimality of a submanifold is a complicated PDE that can be solved only in some special cases. It turned out that the symmetry reduction theory for minimal submanifolds developed by Hsiang and Lawson could give us new examples of minimal S^1-invariant tori and Klein bottles in spheres such that we can investigate extremal spectral properties of their metrics.

Baum-Katz type theorem for exchangeable random sequences

A.S. Rakitko (*Lomonosov Moscow State University*, PROBABILITY, 20 min)

We consider the convergence rates in the law of large numbers for exchangeable random variables in the form proposed by Baum and Katz for independent summands. The exchangeability for a sequence $(X_i)_{i\geq 1}$ of random variables means that, for each $n \in \mathbb{N}$ and any collection $\{k_1, \ldots, k_n\} \subset \mathbb{N}$, the distribution of $(X_{k_1}, \ldots, X_{k_n})$ depends on n and does not depend on $k_1, \ldots, k_n$. Clearly, an i.i.d. sequence of random variables possesses the exchangeability property.

Asymptotic formulae and Riesz basicity of EAF system of Dirac operator with summable potential

A.M. Savchuk (*Lomonosov Moscow State University*, ANALYSIS, 30 min)

Let us consider operator $L_{P,U}$, generated by differential expression

$$l_P(\mathbf{y}) = B\mathbf{y}' + P\mathbf{y}, \quad \mathbf{y} = (y_1, y_2)^t,$$

and boundary condition

$$\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \begin{pmatrix} y_1(0) \\ y_2(0) \end{pmatrix} + \begin{pmatrix} u_{13} & u_{14} \\ u_{23} & u_{24} \end{pmatrix} \begin{pmatrix} y_1(\pi) \\ y_2(\pi) \end{pmatrix}$$

in Hilbert space $\mathbb{H} = L_2[0,\pi] \oplus L_2[0,\pi] \ni \mathbf{y}$ with

$$B = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \qquad P(x) = \begin{pmatrix} p_1(x) & p_2(x) \\ p_3(x) & p_4(x) \end{pmatrix}.$$

We assume that p_j, $j = 1,2,3,4$ are complex valued summable on $[0,\pi]$ functions. The main questions we deal with are asymptotic behavior of eigenvalues and eigenfunctions of $L_{P,U}$ and Riesz basicity of the system of EAF in $\mathbf{H}$.

A probabilistic approach for estimating the Van der Waerden numbers

D.A. Shabanov (*Lomonosov Moscow State University*, PROBABILITY, 45 min)

The talk delas with one of the classical problems of additive combinatorics concerning the Van der Waerden Theorem. This theorem states that for any positive integers n and r there exists the minimum number W(n,r) such that for any N¿W(n,r) every coloring with r colors of the set 1,...,N contains a monochromatic arithmetic progression of length n. We obtain a new asymptotic lower bound for the threshold function W(n,r). The proof is based on a probabilistic technique for colorings of uniform hypergraphs.

Some remarks on Davies uniqueness theorem

A.V. Shaposhnikov (*Lomonosov Moscow State University*, ANALYSIS, 20 min)

We consider the stochastic differential equation $dX_t = dW_t + b(t, X_t)\,dt$, $X_0 = x_0$, where b is a bounded Borel mapping. A.M. Davie showed that for almost all Brownian paths W the associated integral equation has exactly one solution. His proof is quite autonomous (in particular, he does not rely on the uniqueness of strong solutions), but rather technically complicated. It turns out that in many cases the pathwise uniqueness can be proved with a simpler approach. The main idea is to use the Hölder regularity of the flow generated by the strong solution and a modification of the van Kampen uniqueness theorem for ordinary differential equations with a Lipschitz flow and continuous coefficients. This approach also enables us to extend Davie's result to some other classes of non-regular drifts.

A combinatorial model of the Teichmuller metric for surfaces with punctures

V. Shastin (*Lomonosov Moscow State University*, GEOMETRY, 20 min)

We prove that the mapping class group of a punctured surface S equipped with the metric defined by Dynnikov's zipped word length function is quasi-isometric to the thick part of the Teichmuller space of S equipped with the Teichmuller metric.

Eigenvalues asymptotics of differential operators with singular weight. Some applications for small deviations of Gaussian processes

I.A. Sheipak (*Lomonosov Moscow State University*, ANALYSIS, 45min)

As a model problem we study spectral problem for nonhomogeneous string equation with singular mass distribution. In this case the weight function is a distribution from Sobolev space with negative smoothness. The asymptotics of eigenvalues are obtained under the assumption that generalized primitive of weight is an affine self-similar function with integrable square. We introduce self-similar functions of zero and of positive spectral orders. The asymptotics of eigenvalues for these two classes of weights are very different. The differential operators of high order are also considered. As application we find the logarithmic small ball asymptotics for the L_2-norm with respect to a degenerate self-similar measures of a certain class of Gaussian processes including Brownian motion, Ornstein–Uhlenbeck process and their integrated counterparts. We also find the asymptotics of the spectrum for some class of non-affine self-similar weights.

The talk is based on joint works with A.Vladimirov and A.Nazarov.

Some moment estimates for characteristic functions with applications to construction of convergence rate estimates in the central limit theorem

I.G. Shevtsova (*Lomonosov Moscow State University*, PROBABILITY, 45 min)

Some moment estimates for characteristic functions are derived that are applied to construction of moment-type estimates of the accuracy of the normal approximation to distributions of sums of independent random variables and Poisson random sums. The presented estimates for characteristic functions have an untraditional nonlinear dependence on moments and trigonometric dependence on the argument instead of a polynomial one.

Large Deviations for Solutions of Random Difference Equation and Branching Processes in Random Environment

A.V. Shklyaev (*Lomonosov Moscow State University*, PROBABILITY, 30 min)

Let (A_n, B_n) be a sequence of i.i.d. random vectors in $\mathbb{R}^2$ with $A_n > 0$, $B_n > 0$ a.s. Consider a random recurrent equation $Y_{n+1} = A_n Y_n + B_n$ and a random walk S_i with steps $\ln A_i$. Assume that $\mathsf{E}A_n^h$ is finite for some $h > 0$, $\mathsf{E}B_n^h$ and $\mathsf{E}Y_0^h$ are finite for any $h > 0$. It is shown that $\mathsf{P}(Y_n \geq \exp(\theta n)) \sim c\mathsf{P}(\max_{i \leq n} S_i \geq \theta n)$ for some $c > 0$ and any $\theta > \mathsf{E}\ln A_n$, as $n \to \infty$. This result is applied to the study of large deviations of branching processes in random environment.

Polygonal billiards with one-sided scatterings

A. Skripchenko (*Lomonosov Moscow State University*, GEOMETRY, 30 min)

We consider billiards in a polygon (not necessarily rational) with a one-sided scatterings wall. If a particle hits the transparent side of the wall it keeps going straight, and if it hits the reflecting side or the boundary of the table it bounces according to the standard geometrical optics rule. We will discuss the typical behavior of such billiards, their differences from standard ones, their ergodic properties, and some estimates for the combinatorial complexity of the trajectories. (joint work with S.Troubetzkoy)

On the Lojasiewicz-Simon inequality

Adrian Spener (*Ulm University*, GEOMETRY, 30 min)

In 1963, Stanislaw Lojasiewicz used a gradient inequality for analytic functions $\mathcal{E} : U \to \mathbb{R}$ defined on open subsets $U \subset \mathbb{R}^d$ to analyze the asymptotic behavior of global solutions of the corresponding gradient system $\dot{u} = \nabla\mathcal{E}(u)$. In this short talk we will discuss the infinite-dimensional generalization (the so called Lojasiewicz-Simon inequality) and consider its application to the flow of some geometric energy functional.

This work is supported by the DAAD, Strategic Partnership Program.

Nonparametric estimation of the characteristics of stationary Lévy random fields

Evgeny Spodarev (*Ulm University*, PROBABILITY, 50 min)

Consider a stationary real-valued infinitely divisible random field $X = \{X(t), t \in \mathbb{R}^d\}$ with spectral representation

$$X(t) = \int_{\mathbb{R}^d} f_t(x) M(dx), \quad t \in \mathbb{R}^d,$$

where f_t is a known piecewise constant kernel function which is integrable in a proper sense and M is an infinitely divisible independently scattered random measure. We propose different methods for the estimation of the characteristic Lévy triplet of M (shift and scale parameters as well as Lévy density) out of the observations of X. This inverse problem is obviously ill-posed. We discuss the issues of consistency and robustness of different estimation procedures.

This work is supported by the DAAD, Strategic Partnership Program.

Modelling and analyzing dependence structures of random vectors with copulas

Ulrich Stadtmüller (*Ulm University*, PROBABILITY, 50 min)

In this talk an introduction into modelling of dependence structure of random vectors with the help of copulas will be given. On one hand basic facts about copulas and examples of parametric families of copulas are given and on the other hand some results about the statistical Analysis will be presented. Basic material can be found in the books by Nelsen (2006) and Joe (1996).

This work is supported by the DAAD, Strategic Partnership Program.

Semistable and regular models of curves over local fields

Christian Steck (*Ulm University*, GEOMETRY, 20 min)

A crucial ingredient in our computation of Hasse-Weil L-functions (reported on in Michel Börner's talk) is the computation of the **stable model** of the curve in question at its primes of bad reduction. In this talk we will discuss how one can use the stable model to compute a **regular model** as well. This is related to the resolution of quotient singularities of arithmetic surfaces.

This work is supported by the DAAD, Strategic Partnership Program.

Lévy-driven CARMA Processes

Robert Stelzer (*Ulm University*, PROBABILITY, 50 min)

We present an outline of the theory of certain Lévy-driven, multivariate stochastic processes, where the processes are represented by rational transfer functions (Continuous-time AutoRegressive Moving Average or CARMA models) and their applications in non-Gaussian time series modelling. We discuss in detail their definition, their spectral representation, the equivalence to linear state space models and further properties like the second order structure and the tail behaviour under a heavy-tailed input. Furthermore, we study the estimation of the parameters using quasi-maximum likelihood estimates for the auto-regressive and moving average parameters, as well as how to estimate the driving Lévy process.

This work is supported by the DAAD, Strategic Partnership Program.

Embedding of Riemannian metrics with negative curvature in 3-dimensional Euclidean space

Frédéric Stoffers (*Ulm University*, GEOMETRY, 30 min)

In this talk we solve an embedding problem of a Riemannian metric of class C^4 with negative Gauß curvature subject to the initial condition that the realizing surface in 3-dimensional Euclidean Space may contain a given curve satisfying some compatibility conditions. We proceed by using the Heinz-Lewy-method of introducing by a parameter transformation asymptotic parameters in the second fundamental form of an a priori given surface and solving the corresponding Darboux system for the inverse of such a transformation subject to appropriate initial conditions which shall be determined by using the theory of stripes, a generalization of the ordinary theory of curves in 3-dimensional space in which one pays attention to the curvature of the curve relative to a certain normal field.

This work is supported by the DAAD, Strategic Partnership Program.

Reidemeister classes and twisted inner representations

E.V. Troitsky (*Lomonosov Moscow State University*, GEOMETRY, 45 min)

The study of the structure and counting of Reidemeister classes (twisted conjugacy classes) of an automorphism $\phi : G \to G$, i.e. classes $x \sim gx\phi(g^{-1})$, is closely related to the study the twisted inner representation of a discrete group G, i.e. a representation on $\ell^2(G)$ corresponding to the action $g \mapsto xg\phi(x^{-1})$ $(x, g \in G)$ of G on itself. We study here twisted inner representations from a more general point of view, but the questions under consideration are still close to the important relations to Reidemeister classes. (Joint work with Alexander Fel'shtyn and Nikita Luchnikov).

Properties of Sets Admitting Continuous ϵ-Selections

I.G. Tsarkov (*Lomonosov Moscow State University*, ANALYSIS, 45 min)

We say that $\varphi : X \to M$ is an ϵ-selection for a subset M of a Banach space X if $\|\varphi(x) - x\| \leq \rho(x, M) + \epsilon$, where $\rho(x, M) = \inf\limits_{y \in M} \|x - y\|$.

We shall be concerned with sets admitting continuous ϵ-selection. In particular, it is shown that a bounded infinity connected subset M of a Banach space X admits a continuous ϵ-selections for any $\epsilon > 0$.

Steiner ratio problem and infinite boundaries

A.A. Tuzhilin (*Lomonosov Moscow State University*, GEOMETRY, 50 min)

The Steiner ratio of a metric space is a non-trivial numerical characteristic of the space appeared in the context of the famous Steiner problem. At present time, this value is known only for a few metric spaces. In spite of that the Steiner ratio is defined in terms of finite subsets of the metric spaces, it seems that the real progress may be attained only after extending the theory to infinite boundaries. We discuss some generalizations of such type. The talk is devoted to the memory of Professor Dietmar Cieslik (1954-2012) who was the leading specialist in Steiner ratio theory.

Macro- and microscopic structures of the family tree for an island model of branching processes

V.A. Vatutin (*Steklov Mathematical Institute of RAS*, PROBABILITY, 45 min)

A decomposable critical Galton-Watson branching process with N types of particles labelled $1, 2, ..., N$ is considered in which a type i parent may produce individuals of types $j \geq i$ only. This model may be viewed as a stochastic model for the sizes of a geographically structured population occupying N islands, the location of a particle being considered as its type. The newborn particles of island $i \leq N-1$ either stay at the same island or migrate, just after their birth to the islands $i+1, i+2, ..., N$. Particles of island N do not migrate. We investigate the macroscopic and microscopic structures of the family tree of this process as well as the distributions of the birth moment and the type of the most recent common ancestor of the individuals existing in the population at a distant moment n.

Spectral analysis and representations for the solutions of Volterra integrodifferential equations in Hilbert space

V.V. Vlasov & N. A. Rautian (*Lomonosov Moscow State University*, ANALYSIS, 45 min)

We study Volterra integrodifferential equations with unbounded operator coefficients in Hilbert spaces. These equations represent an abstract form of the GurtinPipkin integrodifferential equation describing the process of heat conduction in media with memory and the process of sound conduction in viscoelastic media and arise in averaging problems in perforated media.

Spectral problems for operator-functions that are symbols of these equations are analyzed. The spectra of the abstract integrodifferential GurtinPipkin equation is investigated. The representations for the solutions of such type equations are obtained.

L-functions of curves over number fields

Stefan Wewers (*Ulm University*, GEOMETRY, 50 min)

L-functions are certain analytic functions generalizing the Riemann zeta function. They are one of the main objects studied in arithmetic geometry. In my talk I will focus on Hasse-Weil L-functions which are associated to algebraic curves over number fields. After a survey on the main open problems in the field I will elaborate on the problem of actually computing the L-function of a given curve and experimentally verifiying certain conjectures.

This work is supported by the DAAD, Strategic Partnership Program.

Spatio-temporal structure of stochastic lattice systems

E.B. Yarovaya (*Lomonosov Moscow State University*, PROBABILITY, 45 min)

Continuous-time branching random walks on multidimensional lattices provide an important example of stochastic multicomponent systems in which the evolutionary processes depends on the structure of a medium and the spatial dynamics. The structure of a medium is defined by the offspring reproduction law at the sources of branching. The dynamics of such processes is usually described in terms of birth, death and walks of particles on the lattice. Such a description covers various applications of branching random walks. One of the principal problems in branching random walk models is a study of the evolution of populations of particles at an arbitrary point of the lattice and on the entire lattice. It will be developed a theory of weakly supercritical branching random walks on multidimensional lattices, whose evolution will be studied by usage of the methods of the spectral theory of operators with multipoint perturbations. In the frame of the proposed model, it will be undertaken the spatio-temporal analysis of particle systems. These results will help to analyze the distribution of the population inside the propagation front of particles for weekly supercritical branching random walk.

The integrated periodogram of a dependent extremal event sequence

Yuwei Zhao (*Ulm University*, PROBABILITY, 30 min)

We investigate the asymptotic properties of the integrated periodogram calculated from a sequence of indicator functions of dependent extremal events. An event in Euclidean space is extreme if it occurs far away from the origin. We use a regular variation condition on the underlying stationary sequence to make these notions precise. Our main result is a functional central limit theorem for the integrated periodogram of the indicator functions of dependent extremal events. The limiting process is a continuous Gaussian process whose covariance structure is in general unfamiliar, but in the i.i.d. case a Brownian bridge appears. We indicate how the developed theory can be used to detect periodic cycles of extremes in a stationary sequence.

This work is supported by the DAAD, Strategic Partnership Program.

Application of the Laplace transform to the geometrical and probabilistic problems

A.M. Zubkov (*Steklov Mathematical Institute of RAS, Lomonosov Moscow State University*, PROBABILITY, 50 min)

The lecture is based on old papers by the author which were not presented at the conferences. We are interested in problems related to geometry and probability theory. Namely, we discuss a) a probabilistic proof of the Cavalieri principle for some n-dimensional polyhedra (along with recent generalization obtained by F.M.Malyshev), b) the isomorphisms between the sets of probabilistic distributions on $[0\infty)^n$ and on $\{0, 1, \ldots\}^n$.