Multiple zeta values.

Special course given **in English** by [E. A. Ulanskii](http://new.math.msu.su/department/number/dw/doku.php?id=ulanskiy), one term, for students of 2-5 years.

This special course covers results from the times of Jacob Bernoulli and Leonard Euler until nowadays. You will find out how one of the most famous classical problems and its magnificent solution have led to appearance of fascinating branch of modern Number theory. You will get to know the proofs of charming theorems and will hear about open problems as modern as with three hundred year history.

Course programm:

1. Zeta values. Basel problem and its solution by Leonard Euler.
2. Roger Apery’s theorem on irrationality of ζ(3).
3. Tanguy Rivoal and Keith Ball’s theorem on irrationality of ζ(2n+1) for infinitely many n.
4. Wadim Zudilin’s theorem on irrationality of at least one of four numbers ζ(5),ζ(7),ζ(9),ζ(11).
5. Closed formulae for zeta values.
6. Multiple zeta values (MZV) and generalized polylogarithms. Weight and length. Classical polylogarithms. Euler formulae for MZV including ζ(2,1)=ζ(3).
7. Closed formulae for MZV and some values of generalized polylogarithms.
8. Standard relations for MZV.
9. Michael Hoffman relations and their connection with standard relations.
10. Integral representations for MZV and generalized polylogarithms.
11. Sum relation for MZV. Duality for MZV.
12. Yasuo Ohno relations for MZV and their connection with sum formula and duality.
13. Transformations -z/1-z and 1-z for generalized polylogarithms.
14. Linear independence of generalized polylogarithms. Algebraic independence of classical polylogarithms.
15. Colored generalized polylogarithms. Connection between different integral representations of generalized polylogarithms.
16. Identities for integrals of hypergeometric type. Consequences for generalized polylogarithms.
17. Linear spaces generated by values of generalized polylogarithms of fixed weight.